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ON THE SHOCK POLARS IN A GAS WITH GENERAL EQUATIONS OF STATE*

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Media with equations of state more general than those for a polytropic gas (satisfying the conditions of a normal gas /1/) are considered. Additional conditions are derived ensuring properties of the shock polars analogous to those studied in depth for the case of a polytropic gas /2/.

1. Formulation of the problem. The equations of state of a gas are given in the form $\varepsilon = \varepsilon(v, p)$, $p = g(v, S)$ (ε is the specific internal entropy, v is the specific volume, $\rho = v^{-1}$ is the density, p is the pressure and S is the entropy). The functions $\varepsilon(v, p)$, $g(v, S)$ are defined for $0 < v < \infty$, $0 < p < \infty$, $-\infty < S < \infty$, are sufficiently smooth and satisfy the conditions for a normal gas /2/

$$\varepsilon > 0,\ g > 0,\ g_S > 0,\ g_v < 0,\ g_{vv} > 0,\ \varepsilon_p > 0 \tag{1.1}$$

The last condition is connected with the positiveness of the temperature T ($T = \varepsilon_p g_S$). In addition we shall assume that the following limit relations hold:

$$\lim_{S\to-\infty} g(v, S) = 0,\ \lim_{S\to\infty} g(v, S) = \infty,\ \lim_{S\to-\infty} \varepsilon(v, g(v, S)) = 0 \tag{1.2}$$
$$\lim_{S\to\infty} \varepsilon(v, g(v, S)) = \infty,\ \lim_{v\to 0} \varepsilon(v, p) = 0$$

We note that by virtue of the fundamental thermodynamic identity the functions $\varepsilon(v, p)$ and $g_v(v, S(v, p))$ (where $S(v, p)$ is found from the relations $p = g(v, S)$) are connected by the equation

$$\varepsilon_v + g_v\varepsilon_p + p = 0 \tag{1.3}$$

It was shown in /3/ that when conditions (1.1), (1.2) hold, the necessary and sufficient condition for the unique solvability of the problem of the collapse of an arbitrary discontinuity (Riemann's problem) for the equations of one-dimensional gas dynamics, reduces to the following condition for the functions ε, g:

$$\varepsilon_p \geqslant 2\varepsilon p\,(p^2 - 2\varepsilon g_v)^{-1} \tag{1.4}$$

The inequality (1.4) represents the necessary and sufficient condition of monotonicity of the one (p, u)–diagrams of the shock waves, i.e. of the curves in the u, p plane defined by the equation

$$u - u_1 = \pm[(p - p_1)(v_1 - v)]^{1/2} \tag{1.5}$$

Here u is the velocity of the gas in one-dimensional motion, u_1, p_1, v_1 are given quantities and $v = v(p, v_1, p_1)$ where the function $v(p, v_1, p_1)$ is found from the Hugoniot adiabatic equation

$$H(v, p, v_1, p_1) = 0,\quad H(v, p, v_1, p_1) = \varepsilon(v, p) - \varepsilon(v_1, p_1) - 2^{-1}(p + p_1)(v_1 - v) \tag{1.6}$$

when the following inequality, weaker than (1.4), holds:

$$\varepsilon_p \geqslant -p\,(2g_v)^{-1} \tag{1.7}$$

In the theory of plane steady flows the shock polars are analogues of the (p, u)-diagrams. The same curves appear in three-dimensional unsteady problems describing the interaction between a shock wave and a rigid wall /4/ or another strong discontinuity. Let us recall the definition of a shock polar in the stationary case. Let $\mathbf{w} = (w^1, w^2)$ be the gas velocity vector in stationary plane flow. We introduce the angle θ by means of the relation $\operatorname{tg}\theta = w^2/w^1$. Then the relations

$$(\mathbf{w} - \mathbf{w}_1)^2 = (p - p_1)(v_1 - v)$$
$$q^2 = q_1^2 - (p - p_1)(v_1 + v)\quad (q = |\mathbf{w}|)$$

on the strong discontinuity (the lower case index 1 refers to the state before the shock, and the quantities without the index to the state behind the shock) yield, as a corollary, the equation of the (θ, p)-polar

$$\sin(\theta - \theta_1) = \pm[(p - p_1)(v_1 - v - v_1^2 q_1^{-2}(p - p_1)]^{1/2} \times \tag{1.8}$$
$$[q_1^2 - (p - p_1)(v_1 + v)]^{-1/2} = \pm(\varphi(p, v_1, p_1, q_1))^{1/2}.$$

In (1.8), as well as in (1.5), $v = v(p, v_1, p_1)$, the quantity q_1 satisfies the inequality $q_1^2 > -v_1^2 g_v(v_1, S_1)$, and we assume here that condition (1.7) holds.

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We note the obvious properties of the (θ, p)-polar: the curve is defined for $(v_1 - v) q_1^2 v_1^{-2} \geqslant (p - p_1) \geqslant 0$, its two branches are symmetrical about the $\theta = \theta_1$ axis. Everywhere along the curve $|\sin(\theta - \theta_1)| < 1$. Indeed, from $\sin^2(\theta - \theta_1) = 1$ it follows that $p - p_1 = q_1^2 v_1^{-1}$, but these values lie outside the domain of definition. Then there exists θ_*, $0 < \theta_* < \pi/2$ such that $|\theta - \theta_1| \leqslant \theta_*$, and the value θ_* is attained (the limit angle of rotation of the vector $\mathbf{w}$ during the passage across the shock).

We know that in a polytropic gas the shock polar has the following properties: A- there exists a unique value p_0 such, that $q^2 = q_1^2 - (p - p_1)(v_1 + v) > -v^2 g_v(v, S)$ when $p < p_0$, which corresponds to supersonic flows behind the front, and $q^2 = q_1^2 - (p - p_1)(v_1 + v) < -v^2 g_v(v, S)$ when $p > p_0$ (subsonic flows behind the front); B- any straight line $\theta = \text{const}$, $|\theta - \theta_1| < \theta_*$ intersects the shock polar at exactly two points, and the value θ_* is attained at a unique value of p; C- the quantity $|\theta - \theta_1|$ increases monotonically as p increases on the supersonic segments of the shock polar.

The problem consists of describing a class of equations of state in which the properties A, B and C hold for the shock polars.

Some of the well-known properties of the Hugoniot adiabatic (1.6) will be used in what follows. We shall denote by f' the derivative of f with respect to p along the Hugoniot adiabatic. When conditions (1.1), (1.2) hold, we have

$$-g_v(v_1, S_1) < (p - p_1)(v_1 - v)^{-1} < -g_v(v, S) \tag{1.9}$$

at points lying on the Hugoniot adiabatic (1.6) for $p > p_1$ (Tsemplen's theorem). When $p < p_1$, the inequality signs are reversed. Moreover,

$$\left(\frac{p - p_1}{v_1 - v}\right)' = \frac{v_1 - v + v'(p - p_1)}{(v_1 - v)^2} > 0 \tag{1.10}$$

The appearance of $v'(p, v_1, p_1)$ in this formula is due to the fact that, in what follows, condition (1.7) is assumed to hold. From (1.6) it follows at once that when $p \geqslant p_1$, we have along the Hugoniot adiabatic

$$v_1 - v < 2\varepsilon p^{-1} \tag{1.11}$$

We note that the functions ε and g are connected by the inequality /3/

$$-2\varepsilon g_v > p^2 \tag{1.12}$$

If condition (1.4) holds, then the right-hand side of (1.5) is monotonic when $p > p_1$ /3/

$$v_1 - v - v'(p - p_1) > 0 \tag{1.13}$$

2. Property A.

Lemma 1. The validity of property A for the shock polar (1.8) with arbitrary parameters p_1, v_1, q_1 $(p_1 > 0,\ v_1 > 0,\ q_1^2 > -v_1^2 g_v(v_1, S(v_1, p_1)))$ is equivalent to the strict monotonic form of the function $2i + c^2$ ($i = \varepsilon + pv$ is the specific enthalpy, $c = (-v^2 g_v)^{1/2}$ is the speed of sound) along the Hugoniot adiabatic with centre at the point (v_1, p_1) when $p > p_1$.

Proof. The monotonic character implies property A, since in this case the quantity

$$q_1^2 - (p - p_1)(v_1 + v) + v^2 g_v = q_1^2 + 2i_1 - 2i - c^2$$

decreases monotonically along the Hugoniot adiabatic as p increases. Let the function $2i + c^2$ take the same values at the points (v_2, p_2), (v_3, p_3) of the Hugoniot adiabatic with centre at the point (v_1, p_1) $(p_1 < p_2 < p_3)$ where the quantity $2i + c^2$ does not decrease on the segment $[p_1, p_2 - \delta]$ (δ is a small positive quantity)(it always increases strictly for small $p - p_1$. Let us put $q_1^2 = (p_2 - p_1)(v_1 + v_2) - v_2^2 g_v(v_2, S(v_2, p_2))$. Since the function increases on the segment $[p_1, p_2 - \delta]$ and is continuous, we have $2i + c^2 q_1^2 > -v_1^2 (g_v)_1$. We assert that the points (v_i, p_i) $(i = 2, 3)$ belong to the domain of definition of the shock polar with such a parameter q_1. Indeed, the inequality

$$(p_i - p_1)(v_1 - v_i)^{-1} < q_1^2 v_1^{-2} = [(p_i - p_1)(v_1 + v_i) - v_i^2 (g_v)_i] v_1^{-2}$$

follows from (1.9). Then we find on the shock polar two "sonic" points and the property A is violated.

Lemma 2. The derivative $(2i + c^2)' > 0$ on any Hugoniot adiabatic for $p \geqslant p_1 > 0$ if and only if the functions $\varepsilon(v, p)$ and $g(v, S(v, p))$ satisfy the inequality

$$\varepsilon_p \geqslant \frac{v^2 \varepsilon g_{vv} + (v - v^2 g_{vS}(2g_S)^{-1})(p^2 + 2\varepsilon g_v)}{-p^2 - 2\varepsilon g_v + v^2 p g_{vv}} \tag{2.1}$$

Note. For the class of equations of state disucssed here condition (2.1) is stronger than (1.7) only on the set of points (v, p), defined by the inequality

$$(2g_v)^{-1} p - v + v^2 g_{vS}(2g_S)^{-1} - v^2 g_{vv}(2g_v)^{-1} > 0 \tag{2.2}$$

(The inequality (2.2) represents the condition of positiveness of the difference of the quantities appearing on the right-hand sides of inequalities (2.1) and (1.7)). In particular, if for any equation of state $p = g(v, S)$ the set of solutions of the inequality (2.2) is empty, then inequality (2.1) holds everywhere by virtue of (1.7).

Proof of Lemma 2. Having determined the derivative $(2i+c^2)'$ we obtain, by virtue of (1.6), the inequality

$$(v_1 - v + (p - p_1) g v^{-1}) \Phi(p, v, p_1, v_1) > 0, \quad \Phi(p, v, p_1, v_1) = -g_v\left(\varepsilon_p + v + \frac{v^2 g_{vv}}{2g_v} - \frac{v^2 g_{vS}}{2g_S}\right) + \frac{v^2 g_{vp}(p_1 - p - 2\varepsilon_p g_v)}{2(p_1 - p - (v_1 - v) g_v)} \tag{2.3}$$

equivalent to the inequality $(2i+c^2)' > 0$. The first cofactor in (2.3) is positive by virtue of (1.1), (1.9), therefore it is sufficient to explain the condition of the positiveness of Φ. Let us fix any point (v, p). The centres (v_1, p_1) of different Hugoniot adiabatics passing through this point lie on the curve $H = 0$. The derivative $d\Phi/dp_1$ on $H = 0$ is positive for fixed v, p and $0 \leqslant p_1 \leqslant p$ by virtue of (1.1), (1.7), (1.9) and the inequality $1 - g_v dv_1/dp_1 < 0$ which follows from (1.9), (1.10) (the quantities in (1.10) with and without an index must be interchanged). Consequently, for Φ to be positive on $H = 0$ at $p_1 > 0$, it is necessary and sufficient that $\Phi(p, v, 0, v + 2\varepsilon p^{-1}) \geqslant 0$, since when $p_1 = 0, v_1 = v + 2\varepsilon p^{-1}$, $\Phi(p, v, p_1, v_1(p_1))$ reaches a minimum in p_1 on $H = 0$. If we write the inequality $\Phi(p, v, 0, v + 2\varepsilon p^{-1}) \geqslant 0$ in equivalent form solved for ε_p, we obtain the inequality (2.1).

3. Property B.

Let condition (1.4) hold.

Lemma 3. Property B is equivalent to a strict increase in the value of $M(p)$ for $p > p_1$

$$M(p) = v_1 (p - p_1) [v_1 + v - v'(p - p_1)] [v_1 - v - v'(p - p_1)]^{-1} \tag{3.1}$$

along the Hugoniot adiabatic with centre at the point (v_1, p_1).

Proof. Differentiating (1.8) we obtain

$$\frac{d\varphi}{dp} = \frac{(1 - v_1 q_1^{-2}(p - p_1))(v_1 - v - v'(p - p_1))}{(q_1^2 - (p - p_1)(v_1 + v))^2} [q_1^2 - M(p)] \tag{3.2}$$

The monotonicity of $M(p)$ implies the property B, since $\varphi' = 0$ at a unique point of the shock polar. Let $M(p)$ take the same values at the points (v_2, p_2), (v_3, p_3) of the adiabatic (1.6) $(p_1 < p_2 < p_3)$. If $M'(p) \geqslant 0$ everywhere, then $M(p) = \text{const}$ for $p \in [p_2, p_3]$, however if a point exists at which $M'(p) < 0$, then p_3 can be chosen so that $M'(p_3) < 0$ and the point p_3 will be close to p_4 where $[p_1, p_4]$ is the largest segment on which $M'(p) \geqslant 0$. We note that the inequality $M'(p) > 0$ is equivalent, by virtue of (3.1), to

$$2v(p - p_1)^2 v'' + (v_1 + v - v'(p - p_1))(v_1 - v + v'(p - p_1)) > 0 \tag{3.3}$$

therefore $M'(p) > 0$ near the point p_1. Let us write $q_1^2 = M(p_3)$. Then $q_1^2 > M(p_1) = -v_1^2 (g_v)_1$ since the points p_3 and p_4 are near each other. The points (v_2, p_2) and (v_3, p_3) belong to the domain of definition of a shock polar with q_1 such, that

$$\frac{p_i - p_1}{v_1 - v_i} = \frac{v_1 - v_i - v'(p_i)(p_i - p_1)}{v_1 + v_i - v'(p_i)(p_i - p_1)} \frac{q_1^2}{v_1(v_1 - v_i)} < \frac{q_1^2}{v_1^2}, \quad i = 2, 3$$

by virtue of (1.10). Property B is violated in the case of a shock polar with such a parameter q_1. In the first case we have $|\theta - \theta_1| = \theta_*$ on the segment $[p_2, p_3]$, and in the second case the derivative φ' changes its sign at least three times. Since $M'(p_3) < 0$, we can find $p_5 < p_3 < p_6$ such that $M(p_5) > q_1^2 > M(p_6)$, but when $(p - p_1)(v_1 - v)^{-1} \to q_1^2 v_1^{-2}$, we have $M(p) > q_1^2$.

Lemma 4. The inequalities (a) or (b) are sufficient for (3.3) to hold

(a) $\quad 0 \leqslant \alpha \leqslant \varkappa^2, \quad \beta \leqslant 4p\varepsilon_t g_v(\varkappa^2 - \alpha)(p^2 + 2\varepsilon g_v)^{-1}$

(b) $\quad \alpha \leqslant 0, \quad \beta \leqslant 4p\varkappa(\varepsilon_t g_v \varkappa - p\alpha)(p^2 + 2\varepsilon g_v)^{-1}$

$$(\alpha = \varepsilon_t g_{\tau S} g_S^{-1} + 2^{-1}, \quad \beta = \varepsilon_{pv} - \varepsilon_t g_{vS} g_S^{-1}, \quad \varkappa = \varepsilon_p g_v p^{-1} + 2^{-1})$$

Proof. The inequality (3.3) can be transformed, by virute of (1.3), (1.6), to the equivalent form

$$N = (p_1 - p + g_v(v - v_1))[\varepsilon_t H_v((v_1 + v) H_v + (p - p_1) H_p) + v(p - p_1)^2 g_v^{-1}\psi] + 2v(p - p_1)^2 \varepsilon_t H_p^2 g_{vv} > 0 \tag{3.4}$$

$$\psi = 2^{-1}(p_1 - p + g_v(v - v_1))\beta + (p_1 - p - 2g_v\varepsilon_p)\alpha$$

Since $(v_1 + v) H_v + (p - p_1) H_p > 2vH_v > 0$, the inequality (3.4) holds for $\alpha \leqslant 0, \beta \leqslant 0$ (when we have $\psi \leqslant 0$). We have the following relation along the curve (1.6) for fixed v, p and varying v_1, p_1:

$$d\psi/dp_1 = \alpha + 2^{-1}(1 - g_v dv_1/dp_1)\beta$$

Repeating the arguments of Sect.2, we conclude that when $\beta > 0, \alpha < 0$, the maximum value of ψ is attained when $p_1 = 0, v_1 = v + 2\varepsilon p^{-1}$, and for $\alpha > 0$ when $p_1 = p$, $v_1 = v$. If $\alpha > 0$, $\beta > 0$, then we have

$$\psi < -2^{-1}(2g_v\varepsilon p^{-1} + p)\beta - 2g_v\varepsilon_p\alpha$$

Inequality (3.3) will hold if

$$2\varepsilon_p\varkappa^2 + (g_v)^{-1} \max_{p_1} \psi \geqslant 0 \tag{3.5}$$

since $H_v((v_1+v)H_v+(p-p_1)H_v) > 2vH_v^2 > 2v\varkappa^2p^2$.

Using the estimates obtained for ψ we obtain, from (3.5), the sufficient conditions a) and b).

We can obtain the sufficient condition for the property B not to hold. It reduces to the requirement that N be negative when $p_1=0$, $v_1 = v + 2\varepsilon p^{-1}$ for some v, p.

4. Property C.

Lemma 5. Let the condition formulated in Lemma 2 hold. Then property C is equivalent to the inequality

$$(p + vg_v)\,\varepsilon_p + pv \leqslant 0 \tag{4.1}$$

Proof. The inequality $\varphi' > 0$ is equivalent, by virtue of the formula (3.2), to

$$[q_1^2 - (p-p_1)(v_1+v) + v^2 g_v](v_1 - v - v'(p-p_1)) + \\ vH_v^{-1}(p - p_1 + g_v(v_1 - v))[(p - p_1 + vg_v)\,\varepsilon_p + v(p-p_1)] > 0 \tag{4.2}$$

The sufficiency of the condition (4.1) for a monotonic increase in $|\theta-\theta_1$ on the supersonic segment of the shock polar follows from (4.2). Indeed, when the inequality (4.1) holds, the quantity ε_p also satisfies inequality (1.4). Then inequality (1.13) holds and the first term of (4.2) is positive on the supersonic segment of the polar. The positiveness of the second term is ensured by the inequalities (1.7), (1.9), (4.1).

We shall now prove the necessity. Let $(v_2, p_2)\,(p + vg_v)\,\varepsilon_p + vp > 0$ at some point. We draw through this point a Hugoniot adiabatic and choose on it a point (v_1, p_1) for sufficiently small p_1, so that

$$((p_2 - p_1 + v_2(g_v)_2)(\varepsilon_p)_2 + v_2(p_2 - p_1) > 0 \tag{4.3}$$

Let us write $q_1^2 = (p_2 - p_1)(v_1 + v_2) - v_2^2(g_v)_2$. Then the point (v_2, p_2) will belong to the domain of definition of the shock polar with parameters v_1, p_1, q_1 (as in Sect.2). By virtue of Lemma 2, the segment of the polar with $p \in [p_1, p_2)$ is supersonic, but we have $\varphi' < 0$ near the point p_2. This violates property C and completes the proof of the lemma.

Note. The monotonic increase in $|\theta - \theta_1|$ as p increases on the supersonic segments of the polar is ensured only by inequality (4.1) (the conditions of Lemma 2 are not used).

As a result we obtain, for the class of general equations of state of a gas satisfying the condition (1.1), (1.2), (1.7), additional conditions equivalent to satisfying the properties A and C for the shock polars in such a gas (Lemma 2 and 5), and sufficient conditions for the property B to hold (Lemma 4).

In connection with the use of shock polars (1.8) in computing the reflections and interactions of shock waves, we can utilize the conditions obtained for a quantitative description of the processes in specific media. For example, if the equations of state of the medium are such that condition (2.1) is violated in some domain of variation of the state parameters, we shall observe, for certain parameters of the flow impinging on the oblique shock, alternation of the supersonic and subsonic modes of flow behind the shock, with the pressure increasing behind the shock. In a gas with equations of state satisfying the conditions of Lemma 4, the problem of homogeneous supersonic flow past a wedge /5/ can have not more than two solutions, just as in the case of a polytropic gas. When the inequality converse to (3.3) is satisfied at the point of some Hugoniot adiabatic, the problem of flow past a wedge can have three or more solutions.

If the equations of state of the gas do not satisfy condition (4.1), then in some domain of the parameters it will be possible for the angle of rotation of the velocity vector in the oblique shock to attain its limit value when the flow behind the shock is supersonic, while in a polytropic gas the limit angle is attained when the flow behind the shock is subsonic. The proofs of the lemmas show how equations of state can be used to determine the parameter domains in which properties A, B and C are violated.

The possible geometrical forms of the shock polars in general two-parameter media were studied in /6/. The properties of the shock polars were juxtaposed in /6/ with the corresponding properties of the Hugoniot adiabatics and with the conditions of the stability of the shock waves. At the same time, other Hugoniot adiabatics were considered which did not project uniquely on to the p axis (violation of condition (1.7)), or did not have the star property relative to the centre (violation of (1.10)). It was also noted that the appearance on the Hugoniot adiabatic of two or more points of inflection may lead to violation of property C. This agrees with the conclusions of the present paper (see Lemma 3) which deals, basically, with the description of a class of two-parameter media with "normal" properties of the shock polars.

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ON NON-ONE-DIMENSIONAL SELFSIMILAR SOLUTIONS WITH PLANE WAVES IN GAS DYNAMICS*

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A set of new exact selfsimilar solutions is obtained, describing non-one-dimensional adiabatic motions of an ideal gas with plane waves. The solutions show a uniform expansion of the gas in planes perpendicular to the direction of the basic motion. The system of equations of gas dynamics is reduced for these solutions to a system of ordinary differential equations /1/. Problems of a short shock and the propagation of a strong detonation wave in a uniformly expanding gas was solved numerically in /2/, where an exact solution was also found for the problem of a short shock for a special value of the adiabatic index.

1. Let us consider the adiabatic motions of an ideal gas whose parameters are given by the formulas

$$\rho = \frac{a}{x^{k+3} t^{s}} R(\lambda), \quad p = \frac{a}{x^{k+1} t^{s+2}} P(\lambda) \tag{1.1}$$

$$v = \frac{x}{t} V(\lambda), \quad v_i = \varepsilon_i^{(\beta)} \frac{x_i}{t}, \quad \lambda = \frac{x}{bt^{\delta}}$$

Here x_i are rectangular Cartesian coordinates $(x_1 = x)$. The velocity components along the x, x_i axes are denoted by v, v_i, and the index i takes the values 2 and 3 (there is no summation over i). The motion (1.1) is assumed to be either two-dimensional $(\beta = 1,\ \varepsilon_2^{(1)} = 1,\ \varepsilon_3^{(1)} = 0,\ v_2 = x_2/t,\ v_3 = 0)$, or three-dimensional $(\beta = 2,\ \varepsilon_2^{(2)} = 1,\ \varepsilon_3^{(2)} = 1,\ v_2 = x_2/t,\ v_3 = x_3/t)$. The constants a and b are of dimensions $[a] = ML^kT^s$, $[b] = LT^{-\delta}$.

The system of equations of gas dynamics reduces, for such motions, to the following system of ordinary differential equations in the variables $z(\tau) = \gamma P/R$, $V(\tau)$, $R(\tau)$, $\tau = \ln|\lambda|$ /1/:

$$\begin{aligned} dz/dV = {} & z\{z[2 - \varkappa(\gamma - 1) - 2V] + [(\gamma + 1)V - 2 + \beta(\gamma - 1)](V - \delta)^2 - (\gamma - 1)V(V - 1)(V - \delta)\}(V - \delta)^{-1} \times \\ & [z(\varkappa - \beta - V) + V(V - 1)(V - \delta)]^{-1} \\ V' = {} & [z(\varkappa - \beta - V) + V(V - 1)(V - \delta)][z - (V - \delta)^2]^{-1} \\ R'(V - \delta) = {} & R[s - \beta + (k + 2)V - V'] \\ \varkappa = {} & [s + 2 + \delta(k + 1)]\gamma^{-1} \end{aligned} \tag{1.2}$$

(γ is the adiabatic index $\gamma > 1$).

The last equation of (1.2) can be replaced by the adiabaticity integral /2/

$$PR^{-\gamma} = \text{const}\,[R(V - \delta)]^{\xi}\lambda^{\eta}$$

$$\xi = \frac{2 - (\gamma - 1)s + \delta[k + 1 - \gamma(k + 3)]}{s - \beta + \delta(k + 2)},$$

$$\eta = -\frac{(\gamma + 1)s + 2(k + 2) + \beta[k + 1 - \gamma(k + 3)]}{s - \beta + \delta(k + 2)}$$

The relations on the shocks are written just as in the case of one-dimensional selfsimilar motions /3/

$$R_1(V_1 - \delta) = R_2(V_2 - \delta) \tag{1.3}$$

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